APPLICABILITY OF A ONE-DIMENSIONAL CONJUGATE SCHEME FOR SOLVING NONSTATIONARY CONVECTION PROBLEMS

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A calculation of the nonstationary temperature of one-dimensional thermal sensors washed by a laminar water flow is performed on the basis of a numerical one-dimensional conjugate scheme. Results of the calculation are compared with experimental data for sensors of different thickness and different material.

In a number of literature sources, in particular, in [1], special attention is given to the necessity of solving conjugate problems when studying convective heat transfer. Despite the fact that this approach does not, as a rule, reveal the physical laws of the process, it makes the search for them substantially easier owing to the possibility of tracing the influence of individual physical parameters and combinations of them on the process. The drawbacks of a general approach to solving conjugate problems are well known: the necessity of solving a system of differential equations (of energy, motion, and continuity) in a three-dimensional statement simultaneously for a liquid and a solid, a large number of nodes in the spatial grid in a numerical solution, the complexity of describing the geometry, and "closure" of the system of equations on the basis of a turbulence hypothesis for turbulent flows.

There are undoubtedly a large number of practical cases where we need not solve the conjugate problem in a general statement and a two- or one-dimensional scheme will suffice. It can be assumed that this is possible when local heat transfer is investigated on bodies of simple geometry (a flat wall, a slot channel, etc.) on condition that there are no considerable velocity or temperature gradients in a direction parallel to the surface. This implies that

$$\frac{\partial t}{\partial y} >> \frac{\partial t}{\partial x} \sim \frac{\partial t}{\partial z}, \quad \frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x} \sim \frac{\partial u}{\partial z},$$

where y is a coordinate reckoned along the normal to the surface.

The heat-transfer coefficient for a laminar boundary layer on a flat plate in a longitudinal flow in accordance with the exact solution is equal to [2]

$$\alpha_x = \frac{\lambda}{2}\sqrt{\left(\frac{u_\infty}{\nu x}\right)}\, f(\mathrm{Pr}), \tag{1}$$

$$f(\mathrm{Pr}) = \left[\int_0^\infty \exp\left(-\mathrm{Pr}\int_0^\xi \zeta d\xi\right) d\xi\right]^{-1},$$

$$\xi = \frac{1}{2}\sqrt{\left(\frac{u_\infty}{\nu x}\right)}\, y, \quad \zeta(\xi) = \frac{\psi}{\sqrt{u_\infty \nu x}},$$

$$u_x = \frac{\partial \psi}{\partial y}; \quad u_y = -\frac{\partial \psi}{\partial x}.$$

In particular, for Pr = 1 the heat flux can be calculated using the equation [3]

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$$q_x = \lambda\,(t_\infty - t_w)\left(\frac{u_\infty}{\nu x}\right)^{1/2}\left(\frac{d\Theta}{d\eta}\right)_{\eta=0}. \tag{2}$$

The value of $(d\Theta/d\eta)$ is presented graphically in [3]. If a numerical value is taken in place of $(d\Theta/d\eta)_{\eta=0}$ and the last factors in (2) are represented as

$$\left(\frac{u_\infty}{\nu x}\right)^{1/2}\left(\frac{d\Theta}{d\eta}\right)_{\eta=0} \cong \frac{1}{3\sqrt{\left(\frac{\nu x}{u_\infty}\right)}} = \frac{1}{\delta_{\mathrm{lam}}}, \tag{3}$$

it is obvious that expression (2) is the Fourier heat conduction equation for a thermal boundary layer (at Pr = 1, $\delta_{\mathrm{lam}} = \delta_{\mathrm{t}}$), i.e.,

$$q_x = \lambda\frac{t_\infty - t_w}{\delta_{\mathrm{t}}}. \tag{2'}$$

This implies that for a steady-state laminar boundary layer the problem of heat transfer on the surface can be solved in two stages: first it is necessary to determine the value of δ_{lam} (and δ_{t} in view of Pr) and then to calculate q_x and α_x by the simplest one-dimensional scheme.

This solution, which is valid for a stationary flow, can be also used for a nonstationary thermal regime if it is assumed that the quasi-stationary approach is valid. Here, when only temperature nonstationarity is involved, the only term that is dependent on time in the right-hand side of expression (1) will be $f(\mathrm{Pr})$. This is connected with the fact that for water Pr depends strongly on the temperature, which changes considerably in time within the region of δ_{t}. As noted in [2], accurate up to 2%, for a laminar boundary layer and Pr = 1

$$\frac{\delta_{\mathrm{t}}}{\delta_{\mathrm{lam}}} = \mathrm{Pr}^{1/3}. \tag{4}$$

As a first approximation we can calculate the Pr number from the wall temperature at the previous step and then determine the value of δ_{t}.

A series of these calculations was performed for a set of one-dimensional thermal sensors different materials and of different thickness: for copper sensors, $h = 25\cdot10^{-3}$, $10\cdot10^{-3}$, and $5\cdot10^{-3}$ m, for aluminum sensors, $h = 25\cdot10^{-3}$ m, $10\cdot 10^{-3}$ m, and for lead sensors, $h = 10\cdot10^{-3}$ m. These thermal sensors were used in performing experiments in [4], which enables us to compare the results of numerical calculations with experiment. Since the hydrodynamic parameters were not measured in the experiments, only a qualitative comparison can be made. This primarily relates to the absence of measurements of the velocity u_∞. As preliminary calculations showed, the value of u_∞ could vary within the range u_∞ = 0.23–0.27 m/sec. In the calculations u_∞ = 0.25 m/sec.

From (3) and (4) it follows that

$$\delta_{\mathrm{t}} = \delta_{\mathrm{lam}}\,\mathrm{Pr}^{1/3} \cong 3\sqrt{\left(\frac{\nu x}{u_\infty}\right)}\,\mathrm{Pr}^{1/3}. \tag{4'}$$

The hydrodynamic parameters were maintained constant in the experiments. Here, however, some fluctuation of the velocity u_∞ (and hence of δ_{lam}) from experiment to experiment is possible, as was already noted. Nevertheless, any variation in u_∞ is small during each experiment, and we may assume that $\delta_{\mathrm{lam}} \cong$ const.

The calculations showed that it is impossible to attain qualitative coincidence of the experimental and calculated curves $t_w(\tau)$, characterizing the rate of change of the sensor surface temperature with time, if we consider that δ_{t} = const. To be more specific, coincidence of these curves was observed only on condition that δ_{t} increase with time according to some law. From the viewpoint of theory this law is represented by expression (4'). Stabilization of δ_{t} late in the transient process (as the temperature head decreased to 20°C) enabled us to estimate

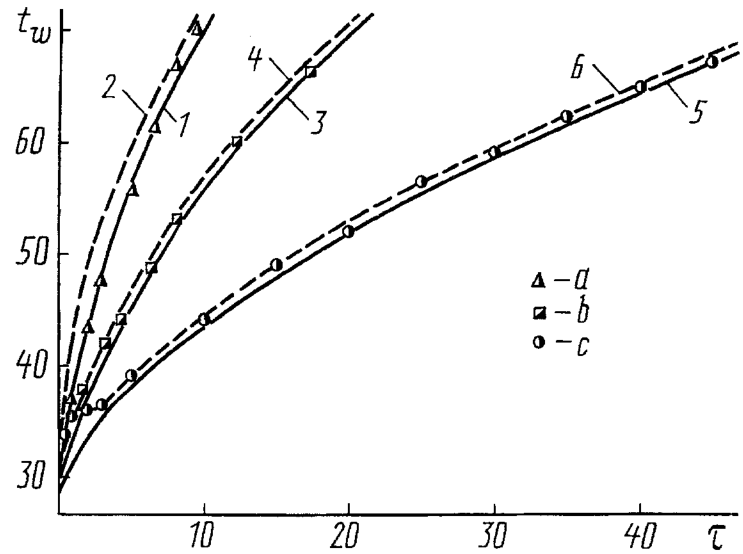

Fig. 1. Plots of $t_w(\tau)$ for copper sensors: the points are the experiment: a) $h = 5 \cdot 10^{-3}$ m, b) $10 \cdot 10^{-3}$, c) $25 \cdot 10^{-3}$; the lines are the calculation: 1, 2) $h = 5 \cdot 10^{-3}$ m; 3, 4) $10 \cdot 10^{-3}$; 5, 6) $25 \cdot 10^{-3}$; 1, 3, 5) calculation according to (4'); 2, 4, 6) calculation taking account of $\delta_{t0} = 0$. t_w, °C; τ, sec.

the value of δ_t and since the value of Pr_w tends to Pr_∞ with time we can estimate the value of δ_{lam}. This value was re-introduced into the calculations, which enabled us to calculate δ_t from relation (4'). Calculations performed for the given law of variation of δ_t yielded good qualitative agreement with experiment (curves 1, 3, and 5 in Figs. 1 and 2).

The thermal sensors in which the rate of heating of the surface was higher ($h \leq 10 \cdot 10^{-3}$ m) displayed particularly good agreement.

For thermal sensors of greater thickness (copper, aluminum, $h = 25 \cdot 10^{-3}$ m) the qualitative agreement was unsatisfactory in the initial period (from 0 to 2.0 sec). The experimental rate of heating of the surface in the initial period was higher than the numerical results. A possible explanation is the influence of the process of immersion of the experimental specimen in the thermostat on the relative velocity of flow along the surface and the value of δ_{lam} and hence δ_t. To take this into account, the authors prescribed the law of variation of δ_t in such a manner that during the initial time interval, corresponding to the time of immersion of the specimen in the thermostat ($\approx$ 1.0–1.5 sec), δ_t increases from 0 to the value corresponding to (4'). The result of a calculation for copper, $h = 25 \cdot 10^{-1}$ m, is given in Fig. 1 (curve 6). It is seen that the experimental data are in good agreement with the calculated ones, even for the instant $\approx$ 1.5 sec, where there is a characteristic bend. Approximately similar results were obtained for an aluminum sensor $25 \cdot 10^{-3}$ m thick (Fig. 2, curve 6). The plots of $t_w(\tau)$ for thinner sensors did not have this characteristic bend and an attempt to take into account the effect of immersion in the numerical calculations resulted in some excess of the calculated temperatures over the experimental ones (curves 2 in Figs. 1 and 2).

As an alternative explanation it can be assumed that with higher heat fluxes on the wall the time of formation of the thermal boundary layer increases.

In the authors' view it is unlikely that in the given case a nonstationarity effect beyond the scope of classical theoretical concepts was manifested: first, the time interval is too small ($\approx$ 1.5 sec), second, there is no manifestation of the effect whatsoever for sensors with $h \leq 10 \cdot 10^{-3}$ m, and third, the fact that $\delta_t = 0$ at the initial instant when the specimen touches the water surface is obvious and the fact that δ_t increases to a value determined by (4') during some finite time is beyond question, the only problem being the duration of this time interval. But in any case it is comparable with the immersion time for the specimen and from the viewpoint of physical assumptions the increase in δ_t during this time is quite real.

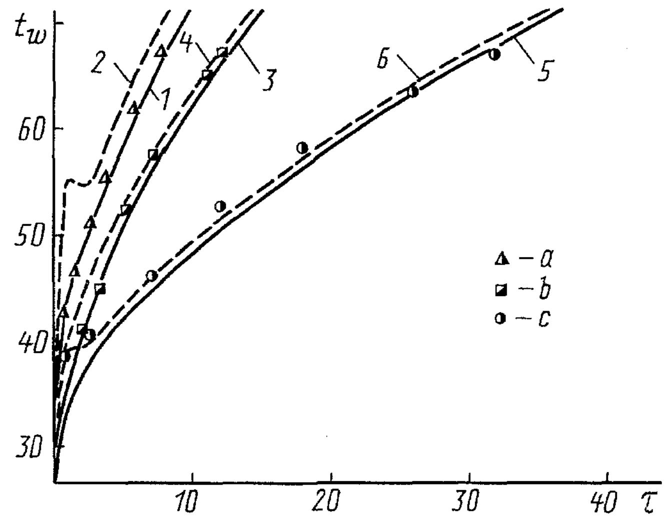

Fig. 2. Plots of $t_w(\tau)$ for lead and aluminum sensors: the points are the experiment: a) lead, $h = 10\cdot10^{-3}$ m, b) aluminum, $h = 10\cdot10^{-3}$ m, c) aluminum, $h = 25\cdot10^{-3}$ m; the lines are the calculation: 1, 2) lead, $h = 10\cdot10^{-3}$ m; 3, 4) aluminum, $h = 10\cdot10^{-3}$ m; 5, 6) aluminum, $h = 25\cdot10^{-3}$ m; 1, 3, 5) calculation according to (4'); 2, 4, 6) calculation taking account of δ_{t0}.

Results relating to the time interval from 1.5 sec to the end of the transient process should be recognized as fairly reliable. The numerical calculations and their comparison with experiment enable us to make the following conclusions:

1. The laws of nonstationary convective heat transfer for the laminar flow of an incompressible liquid along a flat wall correspond to familiar theoretical and numerical solutions for stationary conditions, i.e., a quasi-stationary model is valid. The ratio of the thicknesses of the thermal and hydrodynamic boundary layers is determined by the Pr_w number calculated using the wall temperature rather than the external flow temperature. Apparently, the reason is that q_w in (2), which depends on the temperature gradient near the surface, depends weakly on the temperature distribution and hence on the Pr number in the middle layers and much less in the outer layers of the thermal boundary layer.

2. In many cases of practical importance, solving the conjugate heat transfer problem in a one-dimensional formulation is safficient for calculating local nonstationary heat fluxes and temperatures on condition that δ_t be prescribed correctly.

3. We can recommend an iteration method for engineering calculations of nonstationary heat transfer on a plate in the laminar flow of an incompressible liquid. Here, use should be made of expression (1) as a computational formula. Iterations are necessary because the Prandtl number is not known in advance since the nonstationary temperature of the surface is unknown. As a first approximation either the initial surface temperature or the flow temperature can be taken as the determining temperature for calculating Pr. In subsequent iterations it is necessary to take the current value of the surface temperature in the previous iteration as the determining temperature. The iteration process converges rapidly.

4. Under the same hydrodynamic conditions the wall thickness and its thermophysical characteristics have an effect on the dynamics of variation of the surface temperature in accordance with the laws of heat conduction and hence influence nonstationary heat transfer via the Pr_w number. However, this influence should not be identified with the nonstationarity effect found in [5-7]. The influence of temperature nonstationarity on heat transfer noted in these works has a different physical nature and connected with the strong dependence of the density of the gas flowing along the surface on the temperature.

NOTATION

t, temperature, $^{\circ}$C; x, y, z, coordinates, m; α_x, local heat transfer coefficient, W/(m$^2\cdot{}^{\circ}$C); λ, thermal conductivity, W/(m$\cdot{}^{\circ}$C); u_∞, undisturbed flow velocity, m/sec; ν, kinematic viscosity, m^2/sec; Pr_w, Prandtl number calculated using the surface temperature; Pr_∞, Prandtl number calculated using the undisturbed flow temperature; u_x, u_y, velocity components in the boundary layer, m/sec; q_x, local density of the heat flux, W/m^2; t_∞, undisturbed flow temperature, $^{\circ}$C; t_w, surface temperature, $^{\circ}$C; Θ, dimensionless temperature; η, dimensionless coordinate; δ_{lam}, hydrodynamic boundary layer thickness, m; δ_t, thermal boundary layer thickness, m; δ_{t0}, thermal boundary layer thickness at $\tau = 0$; h, thermal sensor thickness, m; τ, time, sec.

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